Student Decision Making in a Game of Chance and Misconceptions in Probabilistic Reasoning

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This research determined whether a group of 50 Year 9 students playing a card game that involved probabilistic reasoning demonstrated a type of misconception in the selection of strategy they employed. Earlier research into misconceptions in probabilistic reasoning by the author identified widespread use of the heuristics of availability and representativeness by Year 11 students. The present research identified a misconception of a different nature relating to the concept of mathematical expectation.

Background to the Study

In 1990, I reported on observations of student behaviours in a card game that I had made over an number of years teaching Grades 9 and 10 mathematics in Canada (Peard, 1990). The game (Appendix 1) was used in the classroom to introduce elementary concepts in probability. I reported that students demonstrated a preference to take the part of a player over the part of the dealer. I noted further that although the game favoured the dealer if the players do not employ any game strategy, all players were able to develop a simple game strategy which improved the player's chances. The theoretical probabilities under an ideal game strategy were unknown to me at the time, but the fact that in the play of the game many students preferred to take the part of the player, suggested that the odds favoured the player. However, a complete analysis of the game (See Pedler, 1992) showed that the odds are clearly in favour of the dealer, whose probability of winning under the best player strategy is 0.62, and that this figure improves considerably under conditions of imperfect player strategy.

Misconceptions in probabilistic reasoning by young pupils, such as the perceived difficulty of throwing a "six" on a throw of a single die have been well documented. (See Anderson & Pegg, 1988; Green, 1983a; Jones 1974; Pedler, 1977; and Truran, 1992). Misconceptions held by older students, up to 16 years of age, have been documented by Fischbein, Nello, and Marino (1991), Green (1982, 1983b), and Peard (1991, 1993, 1994), and up to university entrance by Shaughnessy (1977, 1981). Misconceptions in probabilistic reasoning by adults have been researched mostly by psychologists rather than mathematics educators. (See Billet, 1986; Kahneman, Tversky, & Slovic, 1983; Scholz 1983, 1991). Most of the misconceptions identified by all of the above researchers can be attributed to some manner of use of either the *availability* or *representativeness* heuristic.

Research Questions

1 Do the students, after playing the game, fail to recognise that the odds favour the dealer? That is to say, do they fail to recognise that the relative frequencies of wins by the player is less than that of the dealer?

2 What criteria do students use to decide on their preference for taking the part of either player or dealer? Do the students demonstrate any type of misconception of probabilities in their decision making?

The Research Sample

The research sample for the study consisted of 51 Year 9 students (two classes) at a metropolitan State high school in the Brisbane region. The prevalence of social gambling at this school had been previously established (Peard, 1991). Both classes had done the Year 9 probability sections in a regular classroom setting. The classes were described by their regular classroom teacher as being heterogeneous in composition and of average achievement.

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Data Gathering

Data were gathered over two 80 minute class periods, towards the end of 1994, when examinations were finished. The game described (Appendix 1) was played by the two classes of Grade 9 students. During the play and at the end of the period, the students answered a research questionnaire in which they gave reasons for their decisions (Appendix 2).

Methodology

1 The classes were informally surveyed to check their familiarity with card games. Without exception, in both classes students were well familiar with the composition of a card deck.

2 The students were arranged into 12 groups of 4 and one group of 3. Each of the students was given a copy of the card game activity (Appendix 1), and the play was explained.

3 Students were then given approximately 20 minutes to play the game using chips for betting, changing dealer each 5 minutes, without further comment or instruction.

4 At the end of this time the students were then given the questionnaire (Appendix 2), and asked to complete questions 1 and 2, using their own informal language.

5 The game was then resumed for a further 20 minutes. At the end of this time the author drew attention to previously taught concepts in basic probability through class discussion and questioning.

After about 10 minutes of teacher-student interaction, in which the students were asked how they could quantify the probability that any particular hand could beat the dealers card, the students were asked to complete Question 3 to 6 of the questionnaire. This section was then corrected before proceeding.

6 Following a discussion of Question 6, students were asked to play the game again for a further 15 minutes, using the strategy of calculating the probability of each hand and betting accordingly using the game strategy: bet maximum if $p>0.5$, bet minimum if $p<0.5$. (The probability of the hand winning was calculated by counting the number of the remaining 48 cards that the hand would beat).

7 Students then completed Questions 7 and 8 of the questionnaire.

Results and Analysis

1 Students had no difficulty in recognising and describing a "good hand." Typical comments included "A good hand has high cards in all suits."

2 With instruction from the author, most students were able to compute the probability that any particular hand would beat the dealer's random card, using the strategy of counting the number of cards that the hand would beat. Some students found difficulties with Questions 3 (iv) and (v).

3 Each class was able to arrive at an answer for Questions 5 and 6.

4 The game strategy of betting the maximum (three chips) when p (as computed using the strategy described) > 0.5 and the minimum (one chip) when p < 0.5 was discussed and seemingly understood by most students.

5 The data from Question 7 were the most important for purposes of answering the research questions posed.

6 The written protocols from Question 7 were discussed with selected individuals in an informal interview.

The responses to Questions 7 and 8 are presented for analysis. Responses to the question of preference (Question 8), dealer, player or no preference were tallied and compared with the students response to their perception of who had the better chance of winning (Question 7).

Table 1 Relationship between student preference and perceived probability

	Preference			
	Dealer	Player	No preference	Total
Greater probability				
Dealer	10	9	0	19
Player	0	0	0	9
Equal	6	3	7	16
Don't know	0	2	5	7
Totals	16	23	12	51

Verbal protocols for the reasons given in Question 8 were summarised into the categories shown.

Table 2: *A summary of reasons for student choice*

Preference		Reasons given	Number of responses
Dealer	(a)	Recognised that dealer has greater probability of winning	10
	(b)	Dealer can win large amount	4
	(c)	No particular reason	2
Player	(a)	Player has greater probability of winning	
	(i)	no particular reason	6
	(ii)	because player has more cards	4
	(b)	Player has greater control over the amount bet*	5
	(c)	Dealer can lose too much*	8
No Preference			
	(a)	Probabilities are equal	6
	(b)	Other/ Reason notgiven	6

* Note: Of these 13, 7 recognised that the dealer had a better probability of winning, but preferred to be the player for the reason given. Three of the respondents in part (a) also made similar comments, but thought that the player had a greater probability of winning.

Conclusions

More than half of the students (32 or 63%) failed to recognise that the odds favoured the dealer. Two possible reasons for this are:

(a) The number of deals was too small for the relative frequency of the dealer's win to be sufficiently close to the theoretical probability of this.

(b) A type of misconception is in operation.

The first of these reasons is rejected by the present author. Each group of students played the game for approximately 40 minutes generating over 100 hands.

An analysis of the first possible reason (using a binomial distribution).

Under the *best* player strategy the probability that the dealer will come out ahead is approximately 0.6 (see Pedler, 1992). So:

p (dealer win) = 0.6
q (player win) = 0.4
n = 100, s.d. = 5
mean no. of dealer wins = np = 60

Thus a result of fewer than 55 dealer wins in 100 deals (one standard deviation below the mean) will occur only about 16 % of the time. We would expect that students in about 2 of the 13 groups (or 8 out of the 51 students) would experience this and fail to recognise the dealer favour. There may be some arguement that a relative frequency of 55 out of 100 would be inadequate for students to recognise favour. However, this may be off-set by the fact that the probability of the dealer winning of 0.6 assumes the *best* player strategy. The probability that the dealer's hand beats that of the players on any one deal is more than 0.7 (See Pedler, 1992), and the actual value of *p* during the play could be considerably higher than 0.6. Since 32 of the 51 failed to recognise this dealer favour it is contended that some other factor is involved. It is therefore hypothesized that a type of misconception is in operation.

Some possible misconceptions. These could relate to:

(i) The fact that the dealer can lose a much greater amount on any one hand than can any of the players was given as a reason for player preference by 8 students. It may be that this masks the dealer's greater long run expectation and leads to the false conclusion that the odds are not in the dealer's favour.

(ii) Five students stated that the player has greater control over the game. This control may be viewed as influencing probability.

(iii) Four students thought that the player had a greater chance of winning (and chose the part of the player) because the player receives more cards.

In none of the above possible explanations can the type of misconception be classified as the use of either the "availability" or "representativeness" heuristic described in the literature as being widespread (See Kahneman, Slovic & Tversky, 1983; Green, 1983b; Peard, 1991, 1993, 1995; Pedler, 1997; Scholz, 1983; Scholz, 1991; and Shaughnessy, 1997, 1981, 1983, 1992).

Possibility (ii) above could be explained simply as a type of probabilistic naivity in which the respondent believes that "control" has an effect on probability. Possibility (iii) above could also be explained as a type of probabilistic naivity, only here the respondent confuses absolute number with probability. However, the first possible misconception requires further consideration. Further evidence that a type of misconception is in operation is provided by the fact that the dealer can also win a much larger amount on any one hand than a player, yet this was given as a reason for prefering to take the part of the dealer by only two students (Table 2). Furthermore, of the 16 who recognised that the odds favoured the dealer, only eight stated a preference to take the part of the dealer. Although not demonstrating a misconception in estimating the probability, these students employed a mathematically incorrect game strategy resulting from their avoidance of situations in which a large loss may result.

The following hypothesis is a generalisation of the situation described in the present study. To the best of the present author's knowledge, following an extensive review of the literature relating to misconceptions in probabilistic reasoning (See Peard, 1994, p. 234), this hypothesis has not been reported elswhere in the literature and it proposed here for further research:

Hypothesis Generated for Further Research

In gambling or betting situations which result in a high probability of a small win and a small probability of a large loss, the player's recognition of the overall positive expectation of the situation is masked by the probability of a large loss.

Further research to test the validity of this hypothesis is currently in progress by the author.

Appendix 1

In groups of 4 to 6, select one person to act as "dealer."

The dealer shuffles the deck of cards and deals four cards to each of the "players" and one card to the dealer.

Players look at their own cards, but not those of other players.

Each player "bets" against the dealer (but not against other players) according to the following conditions:

1. The player wins if he/she holds a card that is the same suit as the dealer's card, but higher ranking (Ace high).

2. The player must bet either one, two, or three of the chips supplied. The player cannot pass.

3. The dealer pays "even money" to each player who wins and collects the bet of each player who loses.

[examples given]

4. After an given number of deals, the dealership is rotated.

Appendix 2

Answer the following questions in your own words:

1. I think a "good hand" is:

2. I think a "bad hand" is:

3. The probability that each of the following hands will beat the dealer's hand is:

(a) Diamond A, Spade Q, Heart 10, Club J

(b) Diamond 2, Diamond A, Heart 10, Club J

(c) Diamond 5, Spade 9, Heart 8, Club 10

(d) Diamond 7, Heart 3, Heart 6, Club J

(e) Club 2, Club 5, Club 8, Club Q.

4. A hand with a 100% probability of winning is:

5. A hand with 0% chance of winning is:

6. How much would you bet on the hands in Questions 4 and 5?

7. Who has the better chance of winning; the player, the dealer, or are they both the same?

8. When you played this game, which part did you prefer to take; the player, the dealer, or did you have no preference?

Write the reasons for your choice.

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